Dirichlet Problems for Laminar Forced Convection with Heat Sources and Viscous Dissipation in Regular Polygonal Ducts

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The effects of viscous dissipation on fully developed laminar forced convection with heat sources in various regular polygonal ducts under the thermal boundary conditions of uniform wall heat transfer per unit duct length and circumferentially uniform wall temperature are approached by the method of point matching. The method uses the exact solutions to the governing partial differential equations and satisfies the boundary conditions exactly only at selected points. This method complements the method of complex variables for various noncircular geometrical configurations where the velocity and temperature fields are not deducible directly from the boundary equations. The relationship between the average velocity and the average of (grad u)^2 over the cross section is pointed out. Other useful relationships between heat transfer quantities based on analogy or the observation of the results from literature and present investigation are also pointed out. The numerical results presented can be used to evaluate the effects of viscous dissipation on mean temperature, overall heat transfer rate at the boundary, heat transfer coefficient and Nusselt number. Graphical results for the representative cases are also presented for the heat transfer rate and the Nusselt number.

The Dirichlet problems for fully developed constant property laminar forced convection with heat sources and constant wall-temperature gradient were formulated by Tao (*1*) using complex variable theory. This method leads to an exact solution for the well-known cases such as the equilateral triangular, elliptical, and cardioid ducts where the velocity and temperature fields are deducible directly from the equations of the boundary curves. This method is not applicable to various noncircular shapes such as square and rectangular ducts. Recently, Tyagi (*2*) extended the above method to the cases with viscous dissipation effect for the same geometrical shapes. The discussed problems without heat sources and viscous dissipation effects were originally solved by Clark and Kays (*3*) for rectangular and equilateral triangular ducts using a relaxation method. The main purpose of this paper is to present an alternate computation-oriented approximate method by point matching for the above problems. As examples, heat transfer results are presented for regular polygonal ducts. The point matching method complements the method of complex variables for the various noncircular ducts where the velocity fields are not deducible directly from the boundary equations. The results of Tyagi (*2*) and present investigation suggest that heat transfer results, such as average temperature, bulk temperature, heat transfer rate, average heat transfer coefficient, and Nusselt number in a given noncircular duct, may be obtained directly for the discussed Dirichlet problems with uniform heat sources and viscous dissipation when the solution for the case of uniform heat input per unit axial length only is available. This will enable one to obtain heat transfer results without actually solving the energy equation involving heat sources and viscous dissipation terms. In practical applications, it is desirable to evaluate the effects of viscous dissipation on the heat transfer rate and the Nusselt number. The present work represents the extension of the earlier work (*4*) taking uniform heat sources and viscous dissipation into consideration.

FORMULATION OF BOUNDARY VALUE PROBLEM

For a steady, fully developed, constant property, laminar flow with heat sources and viscous dissipation in a regular polygonal duct (see Figure 1), the basic momentum and energy equations in polar coordinates are (*1, 2*)

$$\nabla^2 u = \frac{1}{\mu}\frac{dp}{dz} = c_1 \tag{1}$$

$$\nabla^2\theta = \left(\frac{1}{\alpha}\frac{\partial t}{\partial z}\right)u - \frac{Q}{k} - \frac{\mu}{k}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u}{\partial \phi}\right)^2\right] \tag{2}$$

where ∇^2 is the two-dimensional Laplace operator in (r,ϕ). The duct is assumed to be subjected to axially uniform wall heat flux and circumferentially uniform wall temperature at a given axial position. The boundary conditions to be satisfied are

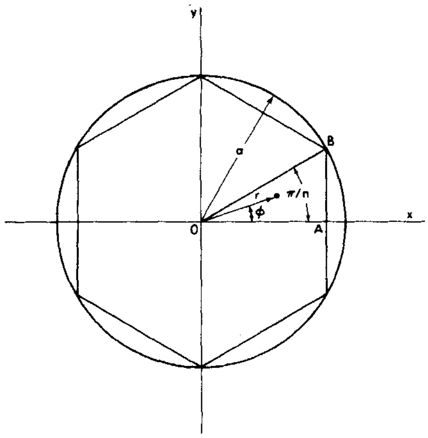

Fig. 1. Coordinate system for n-sided regular polygon.

$$u = \theta = 0 \quad \text{on } \Gamma \tag{3}$$

For regular polygons, because of symmetry, only one-$2n^{\text{th}}$ of the region needs consideration (OAB). It is noted that the local axial temperature gradient $\partial t/\partial z$, which is equal to the mixed-mean axial temperature gradient, is a constant for fully developed flow and should be determined from the net enthalpy change of the flow per unit duct length (*2*). The heat-source intensity Q in general can be a function of position (r, ϕ).

SERIES SOLUTION FOR POINT MATCHING METHOD

The well-known solution of Equation (1) in polar coordinates with due regard for the n-fold symmetry of the problem for regular polygons is

$$u = \left(-\frac{a^2}{\mu}\frac{dp}{dz}\right)\left[-\frac{1}{4}\left(\frac{r}{a}\right)^2 + \sum_{j=0,1,2,3,\ldots}^{\infty} A_j \left(\frac{r}{a}\right)^{jn} \cos jn\phi\right] \tag{4}$$

The above solution already satisfies the boundary conditions of the typical element OAB, $\partial u/\partial \phi = 0$ along $\phi = 0$ and π/n. The undetermined coefficients A_j are to be found from the remaining boundary condition $u = 0$ along AB, $(r/a) = (\cos \pi/n)/\cos \phi$, by the point matching method. In solving the energy Equation (2), a simple case of uniform heat generation will be considered first. Since the problem is linear, the terms representing the usual forced convection, uniform heat generation, and viscous dissipation can be treated separately. With the velocity field available in the form of algebraic-trigonometric polynomial, the differential equation, $\nabla^2\theta_1 = (\partial t/\partial z)\,(u/\alpha)$, can be integrated readily. For the case of uniform heat generation, the governing equation $\nabla^2\theta_2 = -(Q/k)$ is identical to the momentum equation. The mathematical analogy is complete since the boundary conditions $u = 0$ and $\theta_2 = 0$ are also identical. The energy equation for the viscous dissipation effect is

$$\nabla^2\theta_3 = -(\mu/k)\,[(\partial u/\partial r)^2 + (1/r)^2\,(\partial u/\partial \phi)^2]$$

The above equation can be integrated readily since the resulting right-hand side of the energy equation is still in the form of an algebraic-trigonometric polynomial after substituting Equation (4) for the velocity. The complete solution can be obtained by adding the homogeneous solution to the particular solution. The general solution of Equation (2) can then be taken in the following form:

$$\theta = \theta_1 + \theta_2 + \theta_3 \tag{5}$$

where θ_1 is given in reference *4* and θ_2 can be written down by observing the mathematical analogy with the momentum equation and

$$\theta_3 = -\eta\, c_1^{\,2} a^4 \left[\frac{1}{64}\left(\frac{r}{a}\right)^4 + \sum_{j=1,2,3,\ldots}^{\infty} \frac{1}{4} A_j^{\,2} \left(\frac{r}{a}\right)^{2jn} - \sum_{j=1,2,3,\ldots}^{\infty} \frac{(jn)\,A_j}{(4jn+4)} \left(\frac{r}{a}\right)^{jn+2} \cos jn\phi + \sum_{j=2}^{\infty}\sum_{i=1}^{j-1} \frac{1}{2} A_j A_i \left(\frac{r}{a}\right)^{n(j+i)} \cos\{(j-i)n\phi\} - \sum_{j=0,1,2,3,\ldots}^{\infty} D_j \left(\frac{r}{a}\right)^{jn} \cos jn\phi\right] \tag{6}$$

All the unknown coefficients for the temperatures θ_1, θ_2, and θ_3 were determined by the point matching method. Three, five, and ten-point matchings were used along the boundary AB in order to ascertain the convergence of the numerical results. Sufficient accuracy can be obtained by selecting boundary points to be matched at equal angular intervals. The infinite series portion of the solution is truncated so as to include the number of terms corresponding to the number of points to be matched. The resulting system of n linear equations in n unknowns can be solved by a computer using Jordan's elimination method. For brevity, the numerical values of these unknown coefficients will not be tabulated here. It was pointed out (*4*) that the point matching procedure leads to exact solutions for both circular and equilateral triangular ducts. For the case of an arbitrary heat-source function $Q(r,\phi)$, one may attempt to expand $Q(r,\phi)$ into an algebraic-trigonometric series with the unknown coefficients determined by the point matching procedure. Then the resulting equation $\nabla^2\theta_2 = -(Q(r,\phi)/k)$ can be integrated. However, this method remains to be investigated.

HEAT TRANSFER RESULTS

The following heat transfer results, after Tao and Tyagi (*1, 2*), are usually of interest:

$$q = (c_2 u_m - c_3 - \eta\Phi_m)\, kA \tag{7}$$

where

$$\Phi_m = \frac{1}{A}\iint_A \left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u}{\partial \phi}\right)^2\right] r\,dr\,d\phi \tag{8}$$

$$h = -q/S\theta_b = (\eta\Phi_m + c_3 - c_2 u_m)\, kA/S\theta_b \tag{9}$$

and

$$N_{Nu} = h\, D_e/k = 4\,(A/S)^2\,(\eta\Phi_m + c_3 - c_2 u_m)/\theta_b \tag{10}$$

At this point, it is of interest to note that the quantities $c_2 u_m$ and $(-\eta\Phi_m)$ in the expressions for the average heat transfer coefficient h and Nusselt number N_{Nu} are related to each other. This important fact will enable one to obtain $(-\eta\Phi_m)$ immediately from the result of $c_2 u_m$. To prove this, one notes that the velocity u satisfies the following equation:

$$\iint_A \left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u}{\partial \phi}\right)^2\right] r\,dr\,d\phi = -c_1 \iint_A u\,r\,dr\,d\phi$$

or

$$\Phi_m = -c_1 u_m \tag{11}$$

TABLE 1. CONSTANTS

N	M	ℓ_1	ℓ_3	m_1	m_2	m_3	n_1	n_3
3	0.0375	0.200892×10^{-2}	0.100446×10^{-2}	0.301339×10^{-2}	0.0535714	0.136972×10^{-2}	0.0375	0.0375
4	0.0702912	0.681706×10^{-2}	0.341278×10^{-2}	0.973853×10^{-2}	0.0968614	0.441813×10^{-2}	0.0702912	0.0700790
5	0.0888310	1.073068×10^{-2}	0.537079×10^{-2}	1.506556×10^{-2}	0.120675	0.683773×10^{-2}	0.0888310	0.0886010
6	0.0996495	1.340804×10^{-2}	0.670863×10^{-2}	1.867332×10^{-2}	0.134459	0.848031×10^{-2}	0.0996495	0.0994961
7	0.1063656	1.520049×10^{-2}	0.759751×10^{-2}	2.105002×10^{-2}	0.142642	0.956153×10^{-2}	0.106366	0.106449
8	0.1107757	1.646426×10^{-2}	0.823722×10^{-2}	2.276616×10^{-2}	0.148535	1.034416×10^{-2}	0.110776	0.110679
9	0.1138101	1.735198×10^{-2}	0.868142×10^{-2}	2.395261×10^{-2}	0.152369	1.088447×10^{-2}	0.113810	0.113680
10	0.1159796	1.800084×10^{-2}	0.900666×10^{-2}	2.481878×10^{-2}	0.155099	1.127844×10^{-2}	0.115980	0.115813
20	0.1228260	2.013412×10^{-2}	1.007327×10^{-2}	2.767976×10^{-2}	0.163823	1.258283×10^{-2}	0.122826	0.122778
∞	0.125	2.083332×10^{-2}	1.041666×10^{-2}	2.864583×10^{-2}	0.166667	1.302083×10^{-2}	0.125	0.125

Note: The values for $n = 3$ and ∞ (circle) are exact.

The above equation in effect means that mean viscous dissipation is equal to head loss. It is noted that the above relationship is true for both simply and multiply connected ducts. The mean velocity can be expressed as

$$u_m = -M c_1 a^2 \tag{12}$$

Substituting Equations (11) and (12) into Equation (7), one obtains

$$q = -c_4 ka^2 A \left[M + (c_3/c_4 a^2) + M(\eta c_1^2/c_4)\right] \tag{13}$$

Now it is seen clearly that the contribution of viscous dissipation effect $(-\eta\Phi_m)$ in the expression for heat transfer rate can be evaluated directly from the result for $c_2 u_m$. Furthermore, the analogy between momentum and energy equations for uniform heat sources enables one to obtain θ_m immediately from the result for u_m. θ_b for the uniform heat generation case can also be computed readily.

With the velocity and temperature fields available, all the heat transfer results, Equations (7) to (10), can be

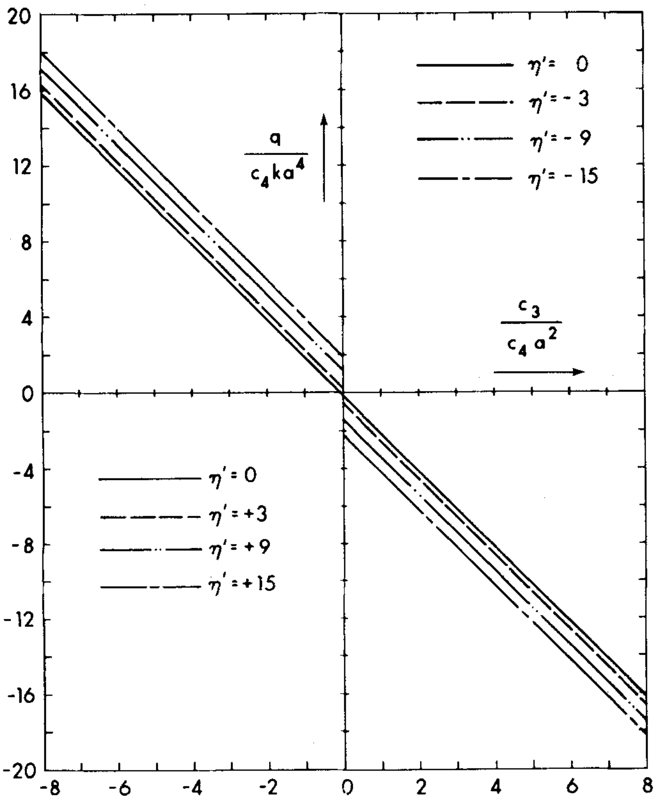

Fig. 2. Heat transfer rate (q/c_4ka^4) vs. heat sources variable (c_3/c_4a^2) with viscous dissipation variable $(\eta' = \eta\ c_1^2/c_4)$ as a parameter in square duct.

computed. The integrations involved were carried out analytically wherever it was possible; otherwise numerical methods were employed. The numerical results for heat transfer can be written in the following form:

$$\theta_m = c_4 a^2 \left[\ell_1 a^2 + \ell_2 (c_3/c_4) + \ell_3 (\eta c_1^2 a^2/c_4)\right] \tag{14}$$

$$\theta_b = c_4 a^2 \left[m_1 a^2 + m_2 (c_3/c_4) + m_3 (\eta c_1^2 a^2/c_4)\right] \tag{15}$$

$$q/(-c_4 ka^4) = \left[n_1 + (c_3/c_4 a^2) + n_3 (\eta c_1^2/c_4)\right] \cdot (n/2) \sin (2\pi/n) \tag{16}$$

$$h = (k/2a) \cos \pi/n \left[\frac{n_1 + (c_3/c_4 a^2) + n_3 (\eta c_1^2/c_4)}{m_1 + m_2 (c_3/c_4 a^2) + m_3 (\eta c_1^2/c_4)}\right] \tag{17}$$

$$N_{Nu} = \cos^2 \pi/n \left[\frac{n_1 + (c_3/c_4 a^2) + n_3 (\eta c_1^2/c_4)}{m_1 + m_2 (c_3/c_4 a^2) + m_3 (\eta c_1^2/c_4)}\right] \tag{18}$$

The constants M, ℓ_1, ℓ_3, m_1, m_2, m_3, n_1, and n_3 appearing in the above equations are listed in Table 1. It is noted that the relation $M = \ell_2$ follows from the mathematical analogy between the momentum equation and the energy equation for uniform heat generation. The relation $n_1 = n_3$ is obvious from Equation (13). The slight difference between n_1 and n_3 is caused by the fact that u_m is based on nine-point matching results, whereas Φ_m is based on ten-point matching results. The difference is insignificant. Although not based on any proof, two interesting observations regarding the numerical values of ℓ_1, ℓ_3 and m_1, m_3, are evident from the results of Tyagi (2) and the present investigation. It is noted that in all cases studied so far $\ell_3 = \ell_1/2$ and $m_3 \simeq (5/11)\ m_1$. For flat duct, which is a limiting case of elliptical duct, $m_3 = (8/17)\ m_1$. The deviation of the factor (8/17) from (5/11) is about 3.5%. This result is noteworthy in view of the difference between a circular duct (or equilateral triangular duct) and a parallel plate channel. The above interesting observations to be further verified by future investigations, coupled with the relations $M = \ell_2$ and $n_1 = n_3$, enable one to evaluate all the heat transfer results for the discussed problems solely from the solutions for forced convection with uniform heat input per unit axial length only.

For the Dirichlet problems in laminar forced convection, the local heat transfer distribution along the circumference of the regular polygonal duct may be of interest. The normal temperature gradient along the duct boundary AB can be computed readily.

Due to analogy, the normal temperature gradient distribution along the boundary due to uniform heat sources only is similar to the shear stress distribution along the same boundary. The normal temperature gradients due to viscous dissipation were calculated along the boundary for various polygons. However, for brevity, these results will not be reported here.

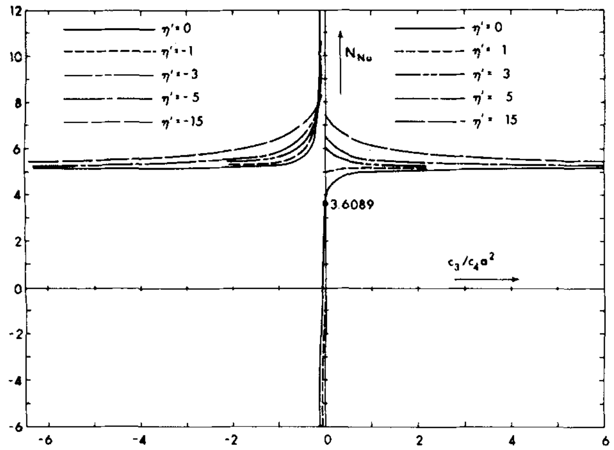

Fig. 3. Nusselt number vs. heat sources variable (c_3/c_4a^2) with viscous dissipation variable ($\eta' = \eta\, c_1^2/c_4$) as a parameter in square duct.

To illustrate the numerical results, both dimensionless heat transfer rate q/c_4ka^4 and Nusselt number N_{Nu} are plotted against the dimensionless heat sources variable c_3/c_4a^2 with the dimensionless viscous dissipation variable $\eta c_1^2/c_4$ as a parameter for the square and hexagonal ducts in Figures 2 to 5 inclusive. To see the physical meaning of the variables involved, one notes that $c_3/c_4a^2 = Q/(\rho c_p/\mu)\ (dp/dz)\ (\partial t/\partial z)/a^2$ and $\eta' = \eta c_1^2/c_4 = (1/\rho c_p)\ (dp/dz)/(\partial t/\partial z)$. For constant property fluid, the viscous

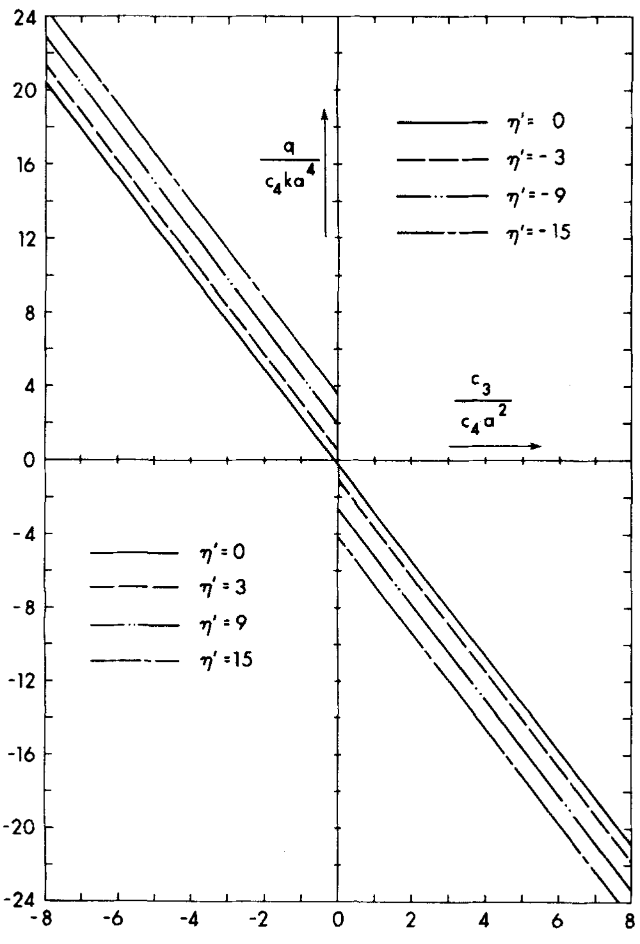

Fig. 4. Heat transfer rate (q/c_4ka^4) vs. heat sources variable (c_3/c_4a^2) with viscous dissipation variable ($\eta' = \eta\, c_1^2/c_4$) as a parameter in hexagonal duct.

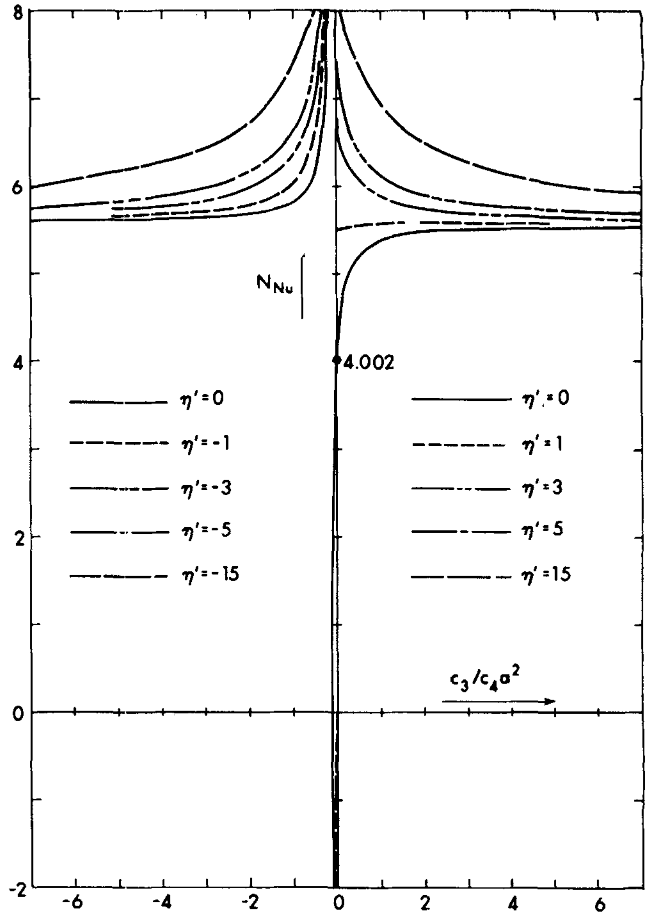

Fig. 5. Nusselt number vs. heat sources variable (c_3/c_4a^2) with viscous dissipation variable ($\eta' = \eta c_1^2/c_4$) as a parameter n hexagonal duct.

dissipation variable η' is seen to be a function of pressure drop and temperature change per unit duct length. The axial pressure gradient dp/dz is always negative. However, the axial temperature gradient $\partial t/\partial z$ can be positive or negative depending on heating or cooling situation. The dimensionless variables used above for heat transfer rate, heat sources, and viscous dissipation (*1*, *2*) are convenient mathematically but these variables are difficult to interpret physically. These dimensionless parameters may be expressed in terms of more familiar physical quantities and the usual dimensionless groups by using Equation (12). The results are

$$
\begin{aligned}
q/c_4ka^4 &= q/(-N_{Pe}/M)(\partial t/\partial z)(ka) \\
c_3/c_4a^2 &= Q/(-N_{Pe}/M)(\partial t/\partial z)(k/a) \qquad (19) \\
\eta c_1^2/c_4 &= N_{Br}/(-MN_{Pe})(a/\Delta z)
\end{aligned}
$$

The last expression was obtained with the local axial temperature gradient $\partial t/\partial z = \Delta t/\Delta z$ where Δz can be interpreted as unit axial length and Δt was used to define the Brinkman number. The results of the heat transfer rates and the Nusselt numbers with various values of the heat sources variable (c_3') and viscous dissipation variable (η') as parameters are plotted against $1/n$ in Figures 6 and 7, respectively.

DISCUSSION AND CONCLUDING REMARKS

The values of the viscous dissipation parameter η' in Figures 2 to 5 are chosen arbitrarily to demonstrate the effect of viscous dissipation. For clarity the range of values, $0 < \eta' < 3.0$, is not shown in the figures. The effects of viscous dissipation on heat transfer rates for $n = 4$ and 6

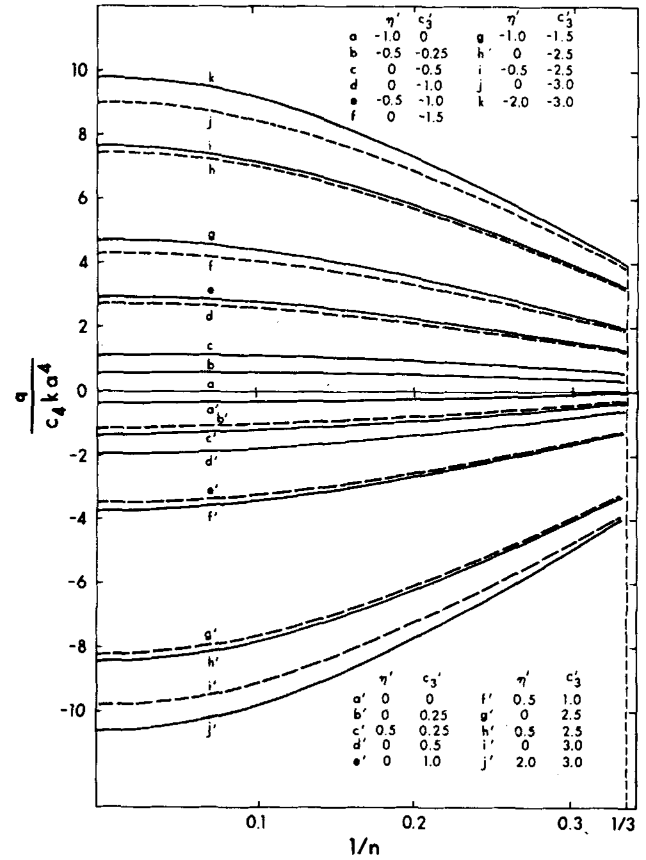

Fig. 6. Heat transfer rate (q/c_4ka^4) vs. $1/n$ with heat sources variable ($c_3' = c_3/c_4a^2$) and viscous dissipation variable ($\eta' = \eta c_1^2/c_4$) as parameters.

are shown in Figures 2 and 4, respectively. It is seen that a linear relationship exists between the heat transfer rate and the heat sources variable for various values of the viscous dissipation parameter. For the case without viscous dissipation effect, a critical point (*1*), $n_1 = -c_3/c_4a^2$, exists beyond which the heat flow may be either toward the fluids (heating) or away from the fluids (cooling). The same situation can happen also for very small values of the viscous dissipation parameter, as is evident from the figures. The critical point can be obtained readily from Equation (16). It is noted that beyond some values of the viscous dissipation parameter, the critical point does not exist. At a given value of the viscous dissipation parameter, the effect of viscous dissipation is independent of the heat sources intensity. For a given value of the heat sources variable, the effect of viscous dissipation increases as the number of sides of regular polygon increases. This conclusion is also evident from Figure 6 where the heat transfer rates are plotted against $1/n$ for various values of the viscous dissipation parameter and the heat sources variable. It can also be said that the effect of viscous dissipation is greater for circular ducts than for noncircular ducts. From Equation (16) it is seen than when $\eta' = -1.0$ and $c_3' = 0$, the heat transfer rate is always zero (see Figure 6). In Figure 6 the effect of viscous dissipation is shown for several representative values of the heat sources variable. The cases with $\eta' = 0$ are shown as dotted lines.

The effects of viscous dissipation on the Nusselt number for $n = 4$ and 6 are shown in Figures 3 and 5, respectively. When the mixed mean temperature θ_b vanishes, the Nusselt number approaches infinity. This happens when $c_3/c_4a^2 = -[m_1/m_2 + (m_3/m_2)(\eta c_1^2/c_4)]$, where both c_3/c_4a^2 and $\eta c_1^2/c_4$ should be negative. As pointed out by Tao (*1*), this does not mean that the heat transfer rate is infinity, since the heat transfer rate is proportional to the product of the heat transfer coefficient and mixed mean temperature. In the case of negative axial temperature gradient, there exists a particular value of the viscous dissipation parameter for which the Nusselt number is independent of the heat sources intensity. This can be found from the equation $d(N_{Nu})/dc_3' = 0$. For example, this occurs when $\eta' = 1.0$ for $n = 20$. When $\eta' = -1.0$ and $c_3' = 0$, both the heat transfer rate and the Nusselt number are zero (see Figures 6 and 7*a*). In the case of cooling (negative $\partial t/\partial z$), without the viscous dissipation effect, curve $d(c_3' = \infty)$ represents the upper limit for the Nusselt number. It is noted that for circular ducts with $c_3' = -0.25$, the Nusselt number is constant (≈ 9.6) for various values of the viscous dissipation parameter. For several cases shown in Figure 7*a*, the effect of viscous dissipation is appreciable. The cases of $c_3' = -0.20$ shown in Figure 7*b* need special attention. For circular ducts with $c_3' = -0.20$ and $\eta' = -3.0$, the Nusselt number is 10.29, whereas with $c_3' = -0.20$ and $\eta' = 0$, the Nusselt number is 16.0. For equilateral triangular ducts, with $c_3' = -0.20$ and $\eta' = -3.0$, the Nusselt number is 5.82 and with $c_3' = -0.20$, $\eta' = 0$, the Nusselt number is 5.28. In other words the effect of viscous dissipation on the Nusselt number is opposite for circular and equilateral triangular ducts when $c_3' = -0.20$. Only a few representative cases are shown in the figures. Accurate quantitative results can be obtained by using Equations (16) and (18). With the exceptions of a few cases involving small negative values of c_3/c_4a^2, in general it can be said that the effect of viscous dissipation increases the Nusselt number. This effect increases as the number of sides of regular polygon increases. The effect is greatest for circular ducts.

The agreements between the numerical results from three-point and ten-point matchings are very good and in

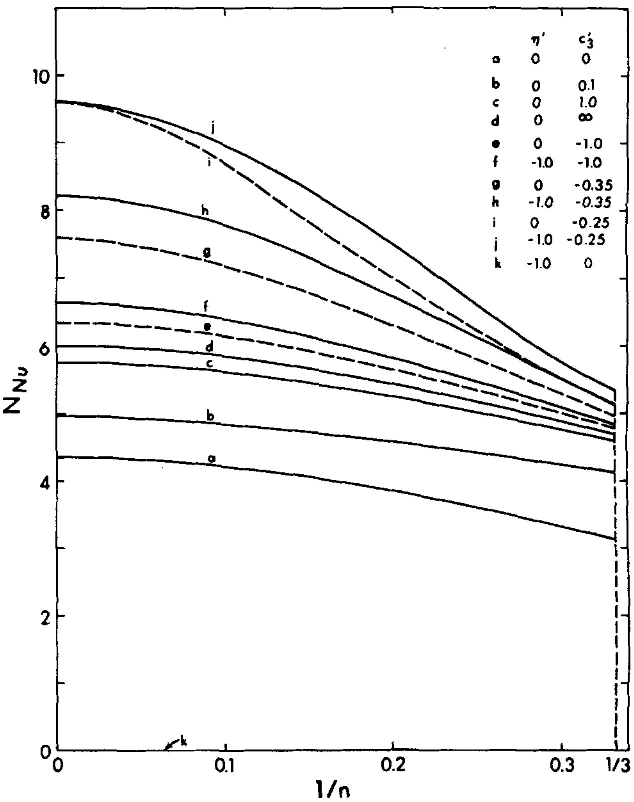

Fig. 7a. Nusselt number vs. $1/n$ with heat sources variable ($c_3' = c_3/c_4a^2$) and viscous dissipation variable ($\eta' = \eta c_1^2/c_4$) as parameters.

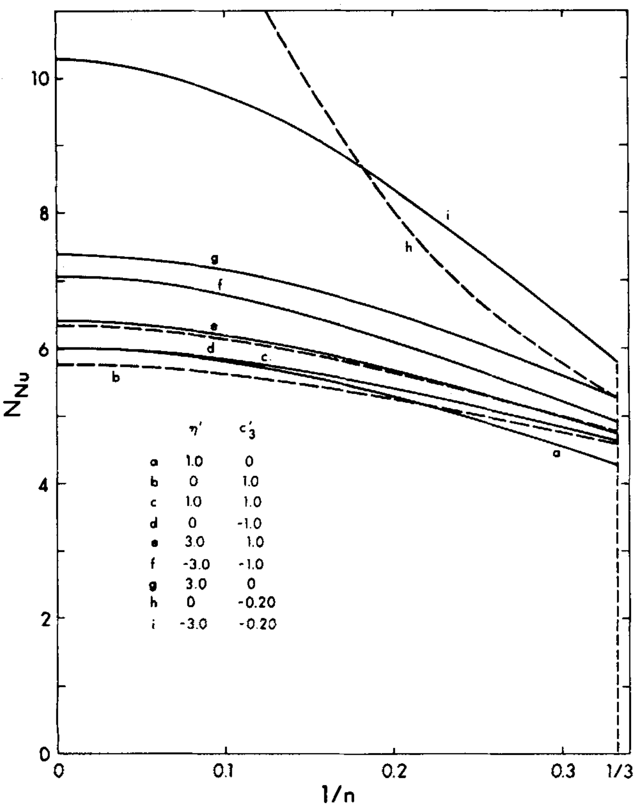

Fig. 7b. Nusselt number vs. $1/n$ with heat sources variable ($c_3' = c_3/c_4a^2$) and viscous dissipation variable ($\eta' = \eta c_1^2/c_4$) as parameters.

most cases the differences are less than 0.5%. Since the boundary conditions are satisfied exactly only at selected points, the boundary errors were also computed for all the cases considered. For the geometrical shapes under consideration, the boundary errors decrease gradually as the number of points matched increases. The maximum boundary error occurs near the corner. For example, for $n = 4$ and ten-point matching, the maximum boundary error for θ_3 is only 0.23×10^{-4} of the maximum temperature at the center of the square duct. Similarly, for $n = 20$ and ten-point matching, the maximum boundary error for θ_3 is 2.2×10^{-4} of the maximum temperature at the center of the duct. A study of the numerical results suggests that all the computed heat transfer results may be well within 1% error of the exact values.

The results of present investigation generally confirm all the findings in references *1* and *2*, indicating that the point matching method can be applied to studies of the discussed forced-convection problems. One notes that the application of point matching method is rather straightforward. However, the applicability of this technique to an arbitrary noncircular geometrical shape cannot be predicted in advance.

For practical application, it is convenient to express the heat sources parameter and viscous dissipation parameter in terms of heat flux q''[B.t.u./(hr.)(sq. ft.)] at the wall and heat sources intensity Q [B.t.u./(hr.)(cu. ft.)] (*2*). Using Equation (12) and the following equation from energy balance

$$\rho c_p u_m \frac{\partial t}{\partial z} = Q + \frac{4q''}{D_e} \tag{20}$$

shows that

$$c_3/c_4 a^2 = -\frac{M}{1 + 4q''/Q D_e} \tag{21}$$

$$\eta c_1^2/c_4 = -\frac{\mu u_m^2}{Ma^2[Q + 4q''/D_e] \cdot g_c J} \tag{22}$$

where $a = 2D_e/\cos \pi/n$ for regular polygon and g_c and J are conversion factors for units.

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NOTATION

A = cross-sectional area
A_j, A_i = series coefficients for velocity
a = radius of circumscribed circle for regular polygon
c_1 = constant parameter, $(dp/dz)/\mu$
c_2 = constant parameter, $(\partial t/\partial z)/\alpha$
c_3 = parameter, Q/k
c_3' = dimensionless parameter, c_3/c_4a^2
c_4 = parameter, c_1c_2
c_p = specific heat at constant pressure
D_e = equivalent hydraulic diameter, $4A/S$
D_j = series coefficient for temperature due to viscous dissipation
h = fully developed heat transfer coefficient
i = integer
j = summation index
k = thermal conductivity
ℓ_1, ℓ_2, ℓ_3 = constants as defined by Equation (14)
M = constant as defined by Equation (12)
m_1, m_2, m_3 = constants as defined by Equation (15)
N_{Br} = Brinkman number, $\mu u_m^2/k\Delta t$
N_{Pe} = Peclet number, $u_m a/\alpha$
N_{Nu} = Nusselt number, hD_e/k
n = number of sides of regular polygon
n_1, n_3 = constants as defined by Equation (16)
Q = heat-source intensity
q = overall heat transfer rate at wall
q'' = heat flux at the duct wall
r = radial coordinate
S = circumference of cross section
t = local temperature
t_w = wall temperature
u = local axial velocity
u_m = average velocity
dp/dz = axial pressure gradient
$\partial t/\partial z$ = axial temperature gradient

Greek Letters

α = thermal diffusivity, $k/\rho c_p$
Γ = boundary curve of cross section
η = parameter, μ/k
η' = dimensionless parameter, $\eta c_1^2/c_4$
θ = temperature variable, $t - t_w$
θ_b = mixed mean temperature
θ_m = average temperature
μ = viscosity
ρ = density
Φ_m = average of $\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u}{\partial \phi}\right)^2\right]$ over the cross section
ϕ = angular coordinate

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